Future Island Universes in a Background Universe Accelerated by Cosmological Constant and by Quintessence

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Abstract

We study bound object formation in a background universe accelerated by cosmological constant and by quintessence. If the acceleration lasts forever, due to the existence of event horizon, one would have naively expected the universe to approach a state of cold death. However, we find that many local regions in the universe can in fact be protected by their own gravity to form mini-universes, provided that their present matter densities exceed some critical value. In the case with cosmological constant (ΛCDM cosmology) the condition of forming mini-universe is that the ratio of present density parameters $\Omega_i^0/\Omega_\Lambda$ ought to be larger than a critical value 3.63. Such mini-universes typically weigh less than 2×10^{14} solar masses, with the lighter ones having tight and compact configurations. In the case with quintessence, the final ratio Ω_i/Ω_q of a mini-universe is found to be always larger than $w_q^2 - w_q$, where w_q is the equation of state parameter.

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I. INTRODUCTION

Recently direct evidences from the studies of Hubble diagram for Type Ia supernovae [1] indicate that our universe is expanding with an increasing rate, i.e., accelerating expansion. This result compels one to seriously consider a dominant component of exotic dark energy, the quintessence, in the cosmic energy balance. Accelerating expansion implies that the deceleration parameter $q = (\Omega_m + (3w_q + 1)\Omega_q)/2$ to be negative, where w_q is given by the cosmic equation of state, $p_q = w_q \rho_q$, of the quintessence Q. As long as $w_q < -1/3$, the corresponding dark energy provides a negative contribution to q, and the relevant range of w_q for acceleration lies in between $-1/3$ and -1. When w_q assumes the extreme value -1, constrained by the weak energy condition [2], the dark energy is reduced to the one arising from the cosmological constant Λ.

The acceleration may stop in the future, i.e., a transient phenomenon with a time-dependent w_q, or may last forever, depending on the nature of the dark energy [3]. A positive cosmological constant, consistent with the supernovae data at $z = 1.7$ [4], leads to a forever acceleration. Quintessence scenario with a constant w_q, not ruled out by data, can also lead to forever acceleration. There are profound implications for a forever accelerating universe [5,6]. The universe will exhibit an event horizon; that is, there exist regions of the universe inaccessible to light probes. It has been argued that such a universe presents a challenge for string theories or any of its alternatives [5]. Many other implications of cosmological constant and quintessence and specific models have been studied in the literature [7–9].

Naively speaking, in a forever accelerating universe, two points at different spatial locations will, as measured at any one of the two points, eventually be pulled apart at the speed of light to approach the future horizon. Hence existing structures will be eternally frozen and the universe will approach a state of cold death. Is this an unavoidable consequence everywhere in the universe, or may some mechanisms be in operation that salvage the future? The present work confronts this issue and shows that the self-gravity of matter can protect a local universe from ending up with cold death, provided that the present local

matter density is sufficiently high. Contrary to the above naive expectation, a large number of mini-universes, including our own Local Group, will become self-bound and survive the destruction of cosmic repulsive force due to the quintessence. They can naturally arise despite that these mini-universes will still be falling into each other's horizon at a sufficiently late time and become isolated island universes.

At present, a variety of dark energy candidates can accommodate the experimental data. If the dark energy arises from a positive cosmological constant, the ΛCDM cosmology, data from recent cosmic microwave background (CMB) radiation [10] further constrain the present energy density parameters to be $\Omega_m^0 = 0.35$ and $\Omega_\Lambda^0 = 0.65$. While for quintessence, depending on the value w_q, the present matter density Ω_m^0 and quintessence density Ω_q^0 can be different [11].

For our purpose of addressing the future of local bound objects in an accelerating universe, the cosmological constant presents the worst scenario since the cosmological constant gives rise to the strongest repulsive force against formation of bound objects. If objects are not torn up by the cosmological constant repulsive force, they will survive in other scenarios.

The paper is arranged as the following. In Sec.(2), we first consider the ΛCDM cosmology in details, by assuming that the present universe is spatially flat with 65% energy given by the cosmological constant ($\Omega_\Lambda^0 = 0.65$) and another 35% by the matter ($\Omega_m^0 = 0.35$). In Sec.(3), we then consider the case where the cosmological constant is replaced by the quintessence with a constant w_q. In both cases the mini-universe can form by self-gravity, but the properties of critically bound mini-universes are very different. Our conclusion is given in Sec.(4).

II. BOUND OBJECT FORMATION WITH COSMOLOGICAL CONSTANT

We shall first adopt a toy model to address the questions of whether bound objects may still form after the background expansion has started to accelerate and how these objects become virialized. We then study a more realistic model where the observation/simulation-

motivated features of virialized objects are taken into account.

A. Self-Binding by Local Gravity

The toy evolutionary model for bound object formation assumes the universe to consist of the background and an isolated cold dark matter fluctuation that grew from a small-amplitude since the early epoch. The localized perturbation possesses spherical symmetry and consists of two concentric solid spheres of different densities. The inner sphere has a radius r_i with a uniform dark-matter density ρ_i, which is greater than the background matter density ρ_m. The outer sphere has a radius r_{out} and a uniform compensating under-density, such that the averaged matter density within r_{out} equals ρ_m. Any particle outside r_{out} feels no extra gravity resulting from the over-density and expands as it would have been in a homogeneous universe. The inner sphere can be regarded as a closed Friedmann universe, which has a Hubble parameter, H_i, given by

$$H_i^2 = \left(\frac{\dot{r}_i}{r_i}\right)^2 = \frac{8\pi G}{3}(\rho_i + \rho_\Lambda) - \frac{\kappa_i}{r_i^2}, \tag{1}$$

where $\rho_\Lambda = (1/8\pi G)\Lambda$ is a constant and κ_i is the curvature of the mini-universe.

A test particle at r_i feels an effective potential $V(r_i)$,

$$V(r_i) = -\frac{GM_i}{r_i} - \frac{4\pi G}{3}\rho_\Lambda r_i^2. \tag{2}$$

Conservation of matter yields a constant mass M_i within r_i with $M_i = 4\pi\rho_i(r_i)r_i^3/3$. The potential $V(r)$ has a maximum at r_{max} determined by $V'(r_{max}) = 0$ with $V(r_{max}) = -4\pi G\rho_\Lambda r_{max}^2$ and $r_{max} = (\rho_i^0/2\rho_\Lambda))^{1/3}r_0$, where the density at present has been denoted as $\rho_i^0 \equiv \rho_i(r_0)$ with r_0 being the present radius of the over-dense sphere. When the test particle reaches r_{max} with a zero velocity, it will be marginally bound. The density ρ_i corresponding to this situation is called the critical binding density, denoted as ρ_{ic}^0 at present, and any local region with density larger than ρ_{ic}^0 will eventually turn around and collapse.

The curvature of the mini-universe can be conveniently obtained at the turn-around,

after which gravitational collapse ensues, by setting $\dot{r_i}^2 = 0$ and equating $-\kappa_i/2$ to the potential at the turn-around. That is,

$$\kappa_i = \frac{8\pi G\rho_\Lambda}{3}(\frac{\rho_i(r_0)}{\rho_\Lambda}\frac{r_0^3}{r_{ta}^3} + 1)r_{ta}^2, \tag{3}$$

where r_{ta} is the turn-around radius. In particular for a critically bound mini-universe, we have $r_{ta} = r_{max}$ and $\kappa_{ic} = \kappa_i(r_{max}) = 8\pi G\rho_\Lambda r_{max}^2$.

The time span needed for the mini-universe to evolve to the present epoch is given by

$$\begin{aligned} t_i &= \int_0^{r_0} \frac{dr_i}{r_i H_i} = \int_0^{r_0} \frac{dr_i}{r_i\sqrt{(8\pi G/3)(\rho_i + \rho_\Lambda) - (\kappa_i/r_i^2)}} \\ &= \frac{1}{\sqrt{6\pi G\rho_\Lambda}} \int_0^1 \frac{dy}{\sqrt{(\rho_i^0/\rho_\Lambda) + y^2 - (3\kappa_i/8\pi G\rho_\Lambda r_0^2)y^{2/3}}}. \end{aligned} \tag{4}$$

When t_i is known, Eq.(4) can be inverted to solve for the present local density ρ_i^0. Thus one needs to obtain information about t_i. To this end we note that the mini-universe should have started to grow from a small amplitude perturbation at the beginning of matter domination around $z = 3 \times 10^3$. Therefore to a good approximation, the evolution time t_i is equal to the background evolution time t_B. The time t_B for the background Friedmann universe to reach the present ratio $\Omega_m^0/\Omega_\Lambda^0$ is

$$t_B = \int_0^{a_0} \frac{da}{\dot{a}} = \frac{1}{\sqrt{6\pi G\rho_\Lambda}} \int_0^1 \frac{dy}{\sqrt{(\Omega_m^0/\Omega_\Lambda^0) + y^2}} = \frac{1}{\sqrt{6\pi G\rho_\Lambda}} \sinh^{-1}(\sqrt{\frac{\Omega_\Lambda^0}{\Omega_m^0}}). \tag{5}$$

Setting $t_i = t_B$, one can determine the present density ratio ρ_i^0/ρ_Λ for a given κ_i.

We consider three cases of interest for the discussions of the density ratio ρ_i/ρ_Λ: i) the critical mini-universe, where the test particle reaches r_{max} with zero velocity and is therefore marginally bound; ii) the case where the over-density is larger than that of case i) and the test particle has a zero velocity and turns around at present; iii) the over-density is even larger so that the turn around occurred at the time when the background matter density ρ_m^{eq} equaled ρ_Λ, a particular epoch prior to which the matter in-fall had been vigorous.

For case i), we have $\kappa_i = \kappa_{ic} = 8\pi G\rho_\Lambda r_{max}^2$. Given the background density parameters $\Omega_m^0 = 0.35$ and $\Omega_\Lambda^0 = 0.65$, Eq.(4) along with Eq.(5) yield the present critical density ratio $\rho_{ic}^0/\rho_\Lambda = 3.63$ ($\Omega_{ic}(r_0) = 2.36$) and $r_0 = r_{max}/1.22$.

We note that over-dense regions with the present density parameter larger than the critical value $\Omega_{ic}(r_0)$ are abundant in the universe. In fact nearly all future bound objects will never reach r_{max} with a turn-around radius r_{ta} smaller than r_{max}. Inserting κ_i of Eq.(3) into Eq.(4), one obtains their present densities $\rho_i(r_0)$ for a given ratio r_{ta}/r_0. For case ii) the mini-universe that is presently turning around, we have $r_{ta} = r_0$, and it is found that $\rho_i^0/\rho_\Lambda = 5.90$ with $\Omega_{i,0} = 3.83$. Any over-dense region with a density greater than this value has already undergone collapse. For case iii), it is found that $\rho_i^{eq}/\rho_\Lambda = 8.55$ with $\Omega_{i,eq} = 5.52$.

Although the toy evolutionary model so far discussed may seem somewhat simplistic, the model nevertheless illustrates the essential physics [12] and supports our contention. From the above discussions, we clearly see that local bound objects can form against the cosmic repulsive force, as long as the local density parameter $\Omega_i(r_0)$ at present is greater than 2.36. This value is at least a factor of several less than the mean density parameter of the Local Group, which includes the Milky Way, the Andromeda galaxy and more than a dozen of smaller galaxies such as the Magellanic clouds. It suggests that the Local Group has defied the cosmic repulsive force and gravitational collapse is underway.

B. Virialization

Almost all bound objects, except for a set of measure-zero, critically bound ones will undergo, or have already undergone, gravitational collapse. It is instructive to further examine the issue of virialization after an object collapses. The virial density found here helps us address deeper questions concerning the internal structures of bound objects to be addressed below. The virialized kinetic energy of a test particle is given by, $< T >= - < \vec{F} \cdot \vec{r} > /2$. For the simplified model under consideration, $< T >$ is given by $< T >= (GM_i/2r_v) - (4\pi G\rho_\Lambda r_v^2/3)$, where r_v is the virialized radius. This leads to

$$-\frac{k_i}{2} =< T > + < V >= -(\frac{GM_i}{2r_v} + \frac{8\pi G}{3}\rho_\Lambda r_v^2). \tag{6}$$

From both energy and mass conservation together with the expression of k_i at the turn-around, eq.(3), with the subscript"0" replaced by "ta", one finds [13]

$$\frac{\rho_{i,ta}}{\rho_\Lambda}(2-\frac{r_{ta}}{r_v})+2(1-2\frac{r_v^2}{r_{ta}^2})=0. \quad (7)$$

Regions with a space curvature κ_i infinitesimally greater than κ_{ic} of case i) will finally collapse in the infinite future, and we shall also regard this situation as case i) in a broad sense. It is straightforward to obtain that $r_v/r_{ta} = 1/2.73$ for case i), since $\rho_{i,ta}/\rho_\Lambda = 2$ and the corresponding r_v/r_0 is $1/2.24$. The ratio of the virialized density to the Λ energy density is then found to be $\rho_v/\rho_\Lambda = 41$. For cases ii), we find $r_v/r_0 = 1/2.20$ and $\rho_v/\rho_\Lambda = 62$. For case iii), $r_v/r_{ta} = 1/2.13$ and $\rho_v/\rho_\Lambda = 83$.

Case (iii) is interesting from the consideration that the matter in-fall was considerably vigorous prior to the turn-around but became suppressed afterward. Although its collapse started at $z = 0.23$ when $\Omega_m = \Omega_\Lambda$, as the completion of collapse takes a time span twice the turn-around time, this object will be virialized in the future when the background universe reaches $\Omega_\Lambda = 0.89$. By then the background scaling factor $a(t)$ becomes a factor 1.63 times the present value a_0.

After the bound object collapses and forms a virialized object, what does the bound object need to adjust so as to guard itself against the stripping repulsive force? We shall address this question by a model of somewhat more sophistication. It has been empirically found by cosmological N-body simulations, which compute at best up to the present epoch, that cold dark matter particles tend to cluster into individual bound objects with a universal density profile. This density profile has been fitted by the empirical formula:

$$\rho = \frac{\Delta}{r(r_s+r)^2}, \quad (8)$$

where Δ and r_s characterize the mass and inner scale length of the bound object [14]. This density profile has a r^{-1} cusp and a r^{-3} envelope extended out to the virial radius r_v. It is universal in that the parameterized functional form is independent of cosmological parameters and epochs, although the coefficients Δ and r_s are. Outside the virial radius r_v, the density profile must be truncated so that the interior matter can be blended into the background matter. Typically the concentration parameter r_v/r_s exceeds 5 for the present-

day virialized objects, and the r^{-3} envelope in the density profile is noticeable. Such an universal profile has been tested against observations favorably [15].

For our purposes, the universal profile can be extrapolated to describe the configurations of future bound objects. We will be concerned about how the concentration parameter r_v/r_s and the characteristic density Δ vary with the bound-object formation time. Unfortunately, the virial radius r_v is undefined in Eq.(8), therefore prohibiting us from using this density profile directly. One hence needs to seek another method to address the evolution of the universal profile.

The following strategy may be adopted for a quantitative evaluation of the evolution of the profile. The steady-state collisionless dark matter is described by the phase-space distribution function $f(C_1(\mathbf{x},\mathbf{p}), C_2(\mathbf{x},\mathbf{p}), ...)$, where C_i's are the constants of motion. For convenience, we assume the momentum $\mathbf{p}$ to be isotropic and the mass density spherically symmetric. It follows $f(C_i) = f(E)$, a function of only particle energy E. Set $f(E) = 0$ for unbound particles that have $E > 0$. Upon integrating out the momentum from $f(E)(= f(p^2/2m + \phi))$, we obtain the mass density $\rho(\phi)$, a function of the potential ϕ of all forces. The universal profile Eq.(8) is obtained from the Poisson equation for an appropriate $\rho(\phi)$. The functional form of $\rho(\phi)$ can be approximated by the following considerations. The inner singular density profile requires $\rho(\phi) \sim (\phi - \phi_{min})$ near the potential minimum ϕ_{min} at $r = 0$. On the other hand, the outer r^{-3} behavior requires $\rho(\phi) \sim -\phi^3$ at the edge of the potential well, $\phi \to 0$, where the bound particles are about to escape. Thus these desired behaviors can be captured by

$$\rho(\phi) = -\frac{\alpha\phi^3}{\exp[\beta(\phi - \phi_{min})] - 1}. \tag{9}$$

Normalizing all quantities in dimensionless forms, we arrive at the Poisson equation,

$$\frac{1}{x^2}\frac{d}{dx}(x^2\frac{d\psi}{dx}) + 1 = -\frac{g\psi^3}{\exp[b(\psi + 1)] - 1}, \tag{10}$$

where $\psi \equiv \phi/|\phi_{min}|(\leq 0)$, $x \equiv r/L$ with the characteristic length $L \equiv \sqrt{|\phi_{min}|/8\pi G\rho_\Lambda}$, $b \equiv \beta|\phi_{min}|$ and $g \equiv \alpha|\phi_{min}|^3/2\rho_\Lambda$. The gravitational potential ϕ_g on the left-hand side of the Poisson equation has been replaced by $\phi + (4\pi G/3)\rho_\Lambda r^2$.

The solutions to eq.(10) require numerical integration. Different values of g and b yield different Δ and r_s of eq.(8). This is a nonlinear eigenvalue problem, and there exists a locus in the eigenvalue (g, b) space, along which the bound objects satisfy the desired boundary condition that ρ vanishes at the potential maximum. Figure (1) presents the parameter b as a function of g for these bound objects. It is instructive to seek a measure for the the inner r^{-1} core region. Also plotted in Fig.(1) is the dimensionless half-mass-weighted radius $h \equiv \langle xD \rangle_{1/2}$ of these virialized objects, where $D \equiv \rho/2\rho_\Lambda$. The quantity $\langle xD \rangle_{1/2} \equiv 3\int_0^{x_{1/2}} x^3 D dx / x_{1/2}^3$, and $x_{1/2}$ is defined to satisfy $\int_0^{x_{1/2}} x^3 D dx \equiv (1/2)\int_0^{x_{max}} x^3 D dx$ with $D(x_{max}) = 0$. Note that the parameter g varies by a factor of 10^5 and b by several hundred, and yet the quantity h does not change in most parameter space until near the high end of (g, b), where it decreases only by 50%. This result shows that the r^{-1} core contains more than half of the mass for most solutions, except for those with $g \geq 5 \times 10^4$.

Plotted in Fig.(2) are the density and potential profiles of three solutions with $(g, b) = (12, 0.0063), (5 \times 10^4, 2)$, and $(8 \times 10^5, 3.2)$. These solutions, denoted as (a), (b) and (c) respectively, have progressively stronger cores of smaller sizes as both g and b increase. Solution (a), having an extended r^{-1} core, gives no detectable outer envelope in the density profile, whereas solution (c), with a strong and compact core, contains a distinct r^{-3} envelope. These solutions are expected to have progressively larger ratios of ρ_v/ρ_Λ, corresponding to virialized objects formed at progressively earlier epochs.

As virialized objects formed at different epochs appear differently, a future observer ought to be able to date these objects by morphology. To determine the formation epochs, we first start from the extended-core solution (a), which is at the small end of the (g, b) spectrum shown in Fig.(1). It has already been well within a self-similar scaling regime beginning around $(g, b) \sim (196, 0.1)$, where the denominator of $\rho(\phi)$ in Eq.(9) can be expanded to the leading order of small β, yielding $g/b \to 1953$. Different solutions in the scaling regime have the same profiles and hence solution (a) can be identified to be the virialized configuration of the last collapsed objects. Making use of the result derived earlier that $\rho_v/\rho_\Lambda = 41$ within the virial radius x_v for the last bound object, we determine the mass fraction, equal to 0.972,

within the virial radius.

Assuming that the mass fractions, 0.972, within the virial radii of all solutions remain the same, we find that $\rho_v/\rho_\Lambda = 65$ and 85 for solutions (b) and (c), respectively. According to the virialization model analyzed earlier, solutions (b) and (c) correspond to the virialized objects with turn-around times at $z = 0$ and $z = 0.23$, or cases ii) and iii) considered earlier, and they are to complete the collapse in the future when the scaling factors a/a_0 become 2.25 and 1.63, respectively.

We note that the redshift evolution of morphology can be rather rapid for these future virialized objects formed in between $a/a_0 \sim 1.5$ and 4.5; by contrast, the collapsed objects up to the present do not show much difference in morphology, all having a compact core and a r^{-3} envelope.

Common to all virialized objects shown in Fig.(2) is that matter is mostly confined at a considerable distance away from the vacuum radius x_{max}. Moreover, as the potential ψ vanishes quadratically $(x - x_{max})^2$ near the potential maximum, the density should vanish as $(x - x_{max})^6$, according to eq.(9). Using the force balance of pressure against gravity, it means that the local temperature vanishes as $(x - x_{max})^2$ at the edge. That is, the dark matter is cold at the outer edge of the mini-universe. Moreover, conservation of the phase-space volume for collisionless particles demands that $\Delta r^3 \Delta v^3$ remains a constant, and when $\Delta v^3 \to 0$ we have $\Delta r \to \infty$. That is, the mini-universe contains a thick outer layer of cold dark matter particles.

How do these cold particles result from? Some fraction of collapsed dark-matter particles must have been pulled away by the cosmic repulsive force to the background during virialization. Thus, evaporation should serve as an important cooling mechanism to produce a thick outer layer of cold particles. The mass loss due to evaporation may be estimated. We note from Fig.(2) that the cool matter outside the virial radius r_v occupies about 90% of the volume enclosed by x_{max}, but it amounts to only 2.8% of the total mass. The evaporated material can not be much more than the cool matter located in the thick outer layer. Therefore the small mass loss should not create any problem for the above quantitative analyses

that assume mass conservation during virialization.

Finally, the typical masses of bound objects formed at various epochs are also of relevance toward understanding the future world. As the linear power spectrum $P(k)$ is well understood [16,17], this issue can be properly addressed as well. With a given conserved κ_i, the radius r_i can be traced back to a sufficiently high z through Eqs.(4) and (5), thereby fixing the amplitude of linear over-density by the relation $\delta\rho(z) = 3\kappa_i/8\pi G r_i^2(z)$. On the other hand, $\delta\rho(z)$ is also given by $g(z)\int_0^{\pi/r_i(z)} 4\pi k^2 P(k) dk$, where $g(z)$ is the growth factor of linear perturbations [18]. Identification of the two $\delta\rho$'s allows $r_i(z)$ to be determined, and the enclosed mass is simply $M = 4\pi\rho(z) r_i^3(z)/3$, a redshift-independent quantity, as it should. The typical mass of the last bound objects (solution (a) of Fig.(2)) is found to be 2×10^{14} solar masses, the mass of a poor galaxy cluster, and those for solution (b) and (c) are 10^{14} and 4.5×10^{13} solar masses, respectively (assuming $H_0 = 65$ km/sec/Mpc).

III. BOUND OBJECT FORMATION WITH QUINTESSENCE

In this section we study the condition for bound object formation under the repulsive force of the quintessence with a constant w_q. At present experimental data allow for the possibility that the accelerating cosmic expansion be due to quintessence with a relatively wide range of constant $w_q(\geq -1)$ and energy densities [11].

In order to have an accelerating universe at present, the equation-of-state parameter w_q should satisfy

$$w_q < -\frac{1}{3}(1 + \frac{\Omega_m^0}{\Omega_q^0}). \tag{11}$$

Unfortunately, the value of Ω_m^0/Ω_q^0 is still rather uncertain, and its plausible value deduced from data correlates with w_q, with a larger Ω_m^0/Ω_q^0 for a smaller w_q in a flat universe. For example, when $\Omega_m^0 = 35\%$ and $\Omega_q^0 = 65\%$ to make up the energy budget for a flat universe, eq.(11) implies $w_q < -0.5$, but the existing data pushes the plausible w_q to be near -1. Nevertheless, to have an idea as to how the results vary with w_q, we shall consider three representative values, $w_q = -0.5, -0.75$ and -1, in our later discussions.

The quintessence energy density varies with the scaling factor as $\rho_q = \rho_q^0 (a/a_0)^{-3(1+w_q)}$. The reference background cosmological time t_B in this case is given by

$$t_B = \frac{1}{\sqrt{6\pi G \rho_q^0}} \int_0^1 \frac{dy}{\sqrt{\Omega_m^0/\Omega_q^0 + y^{-2w_q}}}. \tag{12}$$

For the study of bound mini-universes, the situation becomes more complicated than that with a cosmological constant because the space curvature κ_i within the collapse region is time-dependent, and eq.(4) is no longer valid. (The analysis presented in Ref. [19], which assumes a constant κ_i, is hence invalid.) This problem should be analyzed using the momentum component of the Einstein equation, which describes the force balance of inertia with gravity and quintessence force. It can be understood as follows. Since the self-gravity of quintessence is negligible, the repulsive force of quintessence at a radius r varies with time as $(a_0/a(t))^{3(1+w_q)}$. This is a time-dependent force, thereby leading to a time-dependent space curvature.

The momentum component of the Einstein equation is a second-order differential equation:

$$\frac{\ddot{r}}{r} = -\frac{4\pi G}{3}[\rho_i + (1+3w_q)\rho_q]. \tag{13}$$

Defining $x = (a/a_0)$, $y = (r/r_0)$, one has

$$\begin{aligned} \dot{x} &= \sqrt{\frac{8\pi G}{3}\rho_q^0}\sqrt{(\Omega_m^0/\Omega_q^0)(1/x) + x^{-(1+3w_q)}}, \\ \ddot{y} &= -\frac{4\pi G}{3}\rho_q^0 y[\frac{\Omega_i^0}{\Omega_q^0}y^{-3} + (1+3w_q)x^{-3(1+w_q)}]. \end{aligned} \tag{14}$$

Combining these two equations one arrives at

$$\begin{aligned} 2\bar{y}''\bar{x}(1+\bar{x}^{-3w_q}) &= \bar{y}'(1+(1+3w_q)\bar{x}^{-3w_q}) \\ &\quad - (\frac{\bar{x}^2}{\bar{y}^2} + (1+3w_q)\frac{\bar{y}}{\bar{x}}\bar{x}^{-3w_q}), \end{aligned} \tag{15}$$

where $\bar{x} \equiv x(\Omega_m^0/\Omega_q^0)^{1/3w_q}$, $\bar{y} \equiv y(\Omega_m^0/\Omega_q^0)^{1/3w_q}(\Omega_m^0/\Omega_i^0)$ and $\bar{y}' = d\bar{y}/d\bar{x}$.

We may solve eq.(15) to obtain the conditions for bound object formation at any background cosmic time t_B, c.f., eq.(12). However, due to the wide range of possible parameters

for the quintessence cosmology, we shall instead concentrate on the critically bound mini-universe. The asymptotic state of the critical mini-universe can be found by letting both $\bar{y} \to \infty$ and $\bar{x} \to \infty$ in the infinite future. The existence of such a state also requires that the background expansion be accelerating, i.e., $w_q < -1/3$. It then follows that eq.(15) has a scaling solution, $\bar{y} = (\Omega_{i\infty}/\Omega_{q\infty})^{1/3}\bar{x}^{(1+w_q)}$, where

$$\frac{\Omega_{i\infty}}{\Omega_{q\infty}} = w_q^2 - w_q, \tag{16}$$

by matching the coefficients in eq.(15). The subscript "∞" refers to evaluation of a quantity at a reference epoch in the infinite future.

The space curvature κ_i for this mini-universe defined as minus of twice the sum of kinetic energy and potential energy (of the force used in eq.(14)) is given, in the asymptotic limit, by

$$\kappa_i = -(\frac{a}{a_\infty})^{-(1+w_q)}(1+3w_q)4\pi G\rho_{q\infty}r_\infty^2, \tag{17}$$

where eq.(16) has been used. This confirms the statement made earlier that with quintessence the curvature is time dependent.

The space curvature decreases with time as $a^{-(1+w_q)}$ but its magnitude cannot be determined from the scaling solution; it can be fixed only by integrating eq.(15) to obtain the entire solution. In this asymptotic state, the absolute values of kinetic, potential and total energies all have the same time dependence with comparable magnitudes. For $w_q = -1$, we recover the cosmological constant case with $\rho_{i\infty}/\rho_{q\infty} = 2$ and κ_i given in eq.(3). The above critical mini-universe is very different from that of the case with a cosmological constant, in that its size grows indefinitely since r grows as $a^{(1+w_q)}$.

One may further extrapolate the ratio $\rho_{i\infty}/\rho_{q\infty}$ back to the present by integrating eq.(15) numerically. The results are shown in Fig. 3. In Fig. 3 we show $\log(\Omega_i/\Omega_q)$ as a function of $\log(\Omega_q/\Omega_m)$ for $w_q = -0.5, -0.75$ and -1. From Fig.3, one can read off the densities of the critically bound mini-universe on the vertical axis for various background densities on the horizontal axis. (The horizontal axis can be regarded as another way for expressing the

background cosmic time, c.f., eq.(12).) Regions whose present local densities exceed by a finite amount above a given curve shown in Fig. 3 will turn around within a finite time and become virialized in a manner similar to that discussed in Sec.(2), though the details differ [20].

However, it is interesting to note that there are differences for the virialized objects between the general quintessence cosmology and the ΛCDM cosmology. For the case with quintessence, a local region with an energy infinitesimally smaller than the critically-bound condition (local matter density larger than the critical density by an infinitesimal amount), the region will undergo collapse in the infinite future. The bound object has a nearly zero binding energy, as can be seen from eq. (17), and hence after virialization its virial radius r_v discussed in Sec.(2) will be infinitely large, a great contrast to the $w_q = -1$ case where the just bound objects have finite sizes.

As has been mentioned earlier in this section, the precise value of Ω_m^0/Ω_q^0 is yet to be determined and the plausible value of Ω_m^0/Ω_q^0 in fact anti-correlates with the value of w_q. Though for $w_q = -1$ the most plausible $\Omega_m^0/\Omega_q^0 = 0.35/0.65$, for $w_q = -0.75$ and -0.5 the most plausible Ω_m^0/Ω_q^0 decreases to approximately 0.1/0.9 and (≈)0/1, respectively [11]. Precise knowledge for the content of energy forms, which can be acquired from more accurate calibration of Type Ia supernovae and measurements CMB radiation as well as other observations [11,21], are crucial for making good use of Fig.3.

IV. DISCUSSIONS AND CONCLUSIONS

In this work, we have quantitatively shown that mini-universes can form against the repulsive force of cosmological constant or quintessence with a constant equation-of-state parameter w_q, as long as they have sufficiently high densities at present. These mini-universes, when virialized, have distinctly different configurations depending on the formation time. Bound objects form earlier appear more compact and are confined by deeper gravitational potentials. This issue is addressed in details for the ΛCDM cosmology, where the mini-

universes are found typically no heavier than a poor galaxy cluster.

As discussed in Sec.(2), the actual size of mini-universe varies, depending on the primordial density power spectrum. It is instructive to examine mini-universes of very large scale. We note that a marginally bound mini-universe has a gravitational radius $r_g^2 = c^2 r_{max}^2/\kappa_i = c^2/(3\Omega_\Lambda H_0^2)$ in the case of Λ-cosmology. If a marginally bound mini-universe has a physical size larger than r_g, no light emitted within can escape from it, and hence behaves like a black hole when observed from the exterior. Such a mini-universe is absolutely stable against the background pulling force. We therefore obtain the minimal size r_{min} of an absolutely stable and marginally bound mini-universe to be $c/\sqrt{3\Omega_\Lambda}H_0$, which is comparable to the Hubble radius of the background universe.

For mini-universes with larger local densities, the required radius is smaller. These mini-universes will eventually undergo catastrophic collapse, and can asymptotically be described by a static space-time metric – the Schwartzschild-de Sitter metric:

$$ds^2 = (1 - \frac{2GM}{r} - \frac{\Lambda}{3}r^2)d^2t - (\frac{d^2r}{1 - 2GM/r - \Lambda r^2/3} + r^2 d^2\theta + r^2 \sin^2\theta d^2\phi). \qquad (18)$$

It has two horizons: an inner horizon at a modified Schwartzschild radius and an outer horizon pertaining to the accelerating cosmic expansion. The region in between the two horizons is unstable, in that an observer located in between the two horizons must eventually fall onto either of them.

Our local universe cannot be within such a closed universe, as the angular scale of the first peak in the CMB anisotropy power spectrum indicates the local universe within the Hubble radius to be spatially flat [10]. This, however, does not preclude the possibility that regions outside the present Hubble horizon may have sufficiently large super-horizon over-densities to become a closed universe. The required present matter over-density Ω_i^0/Ω_m^0 in such universes is as high as 6.75. These regions are very rare if the primordial density fluctuation is indeed scale-invariant, created during the inflation in early universe, with a power spectrum linearly proportional to the wavenumber k. Such local closed universes should be more abundant in the past, and some of them may have been within our present

horizon, perhaps in the form of super-massive black holes.

In the case of quintessence, the situation is more complicated because the mini-universe has a time-dependent space curvature. The curvature actually decreases with time, due to the fact that matter and quintessence as a whole can be regarded as a non-ideal fluid and the local matter over-density gives rise to entropy fluctuations. Space curvature fluctuations pertinent to entropy fluctuations always decrease monotonically in time, regardless whether the fluctuations are of sub-horizon size or super-horizon size. However, adiabatic super-horizon fluctuations can also give rise to curvature fluctuations with no time dependence, for which quintessence fluctuations can grow and collapse as well. They are more relevant for the discussions of the local closed universe in the context of quintessence cosmology. Unfortunately, our analysis presented in Sec.(3) is confined to entropy fluctuations, where the quintessence fluctuation is negligible. Nevertheless, the dynamics of such a local closed universe with a constant space curvature is relatively simple and like that of a homogeneous universe. Thus the discussions given in the preceding paragraph for Λ-cosmology also holds for quintessence-cosmology. The super-massive black hole so formed should contain a substantial amount of dense quintessence field within the Schwartzschild radius, and its asymptotic space-time metric depends on the yet unknown nature of the quintessence field.

Return to the mini-universes arising from entropy fluctuations and of sub-horizon size analyzed in Sec.(3). As long as the matter density is larger than the critical density given in Fig. 3 for the corresponding equation of state parameter, the local region will be dominated by the matter gravity and collapses to become a mini-universe. These mini-universes will become virialized in a similar manner as those of the ΛCDM cosmology [20], due to that virialization depends mainly on the balance between gravity and pressure, and to a lesser degree on the large-scale cosmic pulling force. Despite the similarity, the details can be different. For example, conservation of total energy within the bound object, which is essential for applying the virial condition, no longer holds in quintessence-cosmology, and hence the resulting virial over-density within the object deviates from that of the ΛCDM cosmology. Also the marginally bound mini-universe has an infinitely large virial radius.

At any rate, a single tightly bound mini-universe will, in the future, emerge from the Local Group, whose appearance resembles solution (c) of Fig.(2) in the case of ΛCDM cosmology. Interior to the mini-universe, local physics in the future will remain the same as what we experience presently. However, the mini-universes will become isolated island universes in the future where no communication among them is possible, and each is a lone world. When observed within a given mini-universe, all other mini-universes originally within the horizon will eventually fall onto the horizon and become frozen. Lights emitted from the horizon are frozen as well, making other island-universes dark and invisible.

The detailed properties of the mini-universes, depend on whether the accelerating expansion is due to the cosmological constant or quintessence. New experimental data from various observations in the coming decades are expected to reveal more information. By then we will have a better understanding of the properties of mini-universes. If the acceleration of our universe at present is indeed due to cosmological constant or quintessence with constant w_q, our universe will enter a new era, the era where all island universes are falling onto each other's horizon and appear to fade away. It is, nevertheless, comforting to know that the Milky Way galaxy is in one of these island universes, within each of which most physics remains the same as is today. Review on the details of how an island universe will eventually end itself with physics known today can be found in Ref.[19].

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FIGURES

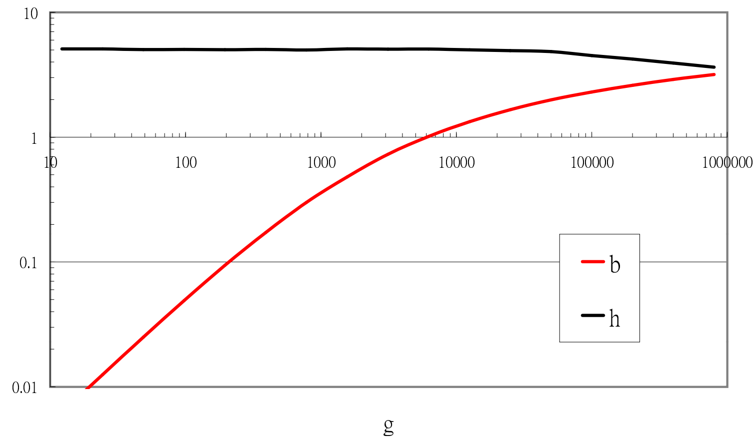

FIG. 1. Nonlinear eigenvalues (g, b) for Eq.(10). When $b < 0.1$, the solutions are in a similarity scaling regime. Also shown is the half-mass weighted radius, h. The constancy of h implies that the r^{-1} core contains more than half of the total mass.

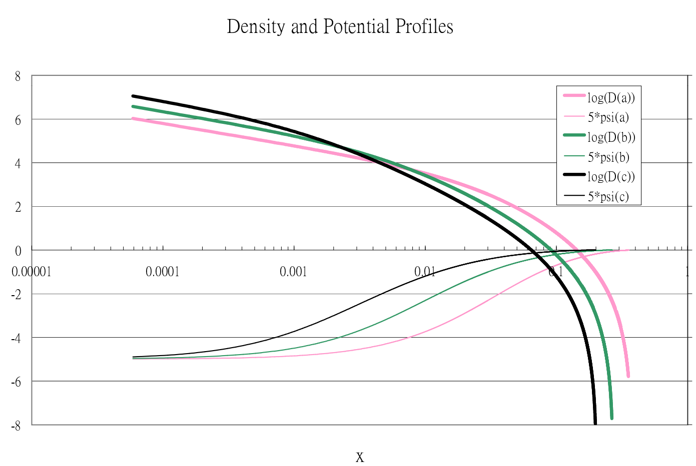

FIG. 2. Dimensionless densities D (in log scale) and potentials ψ for solutions $(a), (b)$ and (c) of Eq.(10).

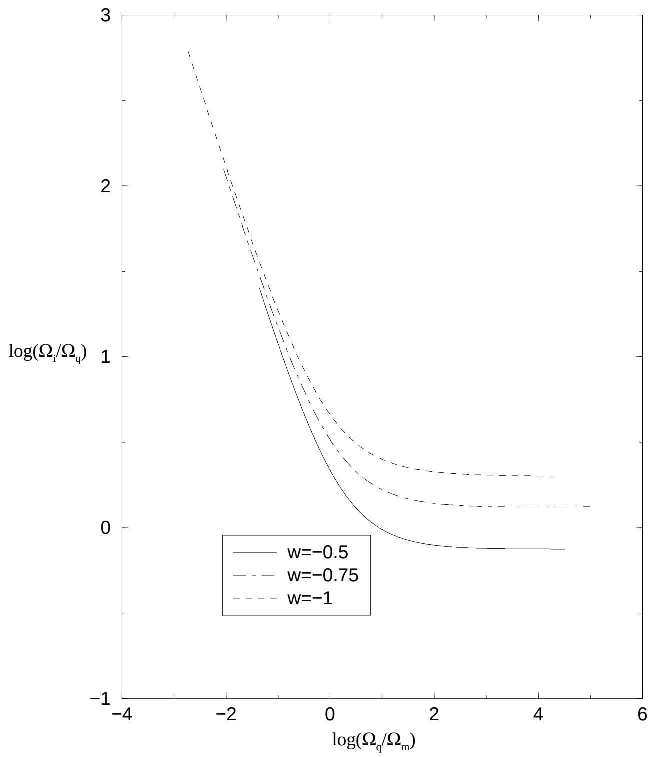

FIG. 3. The barely bound mini-universe critical density as a function of time. The vertical axis is $\log(\Omega_i/\Omega_q)$ and the horizontal axis is $\log(\Omega_q/\Omega_m)$.